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Fluid Dynamics and Transport Phenomena

CFD simulation of solids residence time distribution in a multi-compartment fluidized bed☆

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ABSTRACT

The present work focuses on a numerical investigation of the solids residence time distribution (RTD) and the fluidized structure of a multi-compartment fluidized bed, in which the flow pattern is proved to be close to plug flow by using computational fluid dynamics (CFD) simulations. With the fluidizing gas velocity or the bed outlet height rising, the solids flow out of bed more quickly with a wider spread of residence time and a larger RTD variance (σ^2). It is just the heterogeneous fluidized structure that being more prominent with the bed height increasing induces the widely non-uniform RTD. The division of the individual internal circulation into double ones improves the flow pattern to be close to plug flow.

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1. Introduction

The fluidized bed reactor is characterized by its uniform temperature profile and high reaction efficiency, for it is able to provide a large surface area between gas and solids phases, which leads to a good heat and mass transfer rate between the phases [1–3]. However, the fluidized bed is limited for its non-uniform distribution of solids residence time, so it is undesirable for the production of materials being sensitive to the processing time [4].

For the continuous reactor, the plug flow of solids exhibits the advantage of reducing the irregularity and instability of the bed induced by the gas flow [5–7]. And this can be approached through dividing the bed into multi-compartment by using different types of baffles. Raghuraman and Varma [8,9] used an internal like a baffle or a spiral plate to increase the plug flow tendency of particles. Pydi Setty *et al.* [10,11] found that the increase in the number of stages could reduce the axial mixing and each stage behaved as a single stage unit, with the additional advantage of restricting the bubble growth. Cocquerel [12] showed that the solids flow rate, fluidizing gas velocity and bed aspect ratio had important effects on the tendency to plug flow for the fluidized bed experimentally.

In the literatures mentioned above the main attention lies on the experimental procedure to investigate the regulations of solids flow pattern, but all these modifications increase the complexity of bed, rendering the design and operation more difficult [10]. So far, there have been several types of numerical model to predict the flow behavior of solids, and the axial dispersion model and tanks-in-series model are the two widely used methods to represent the overall spread of solids RTD [13–15]. And they are found to be the useful approaches to describe the physical situation of the fluid system, whereas there usually exists deviation from the real system curve data as the fitting extent mainly depends on the bed geometric characteristic, the size and density of particles, the mean residence time of solids and the fluidizing gas velocity. Morris *et al.* [16] reported a poor agreement between the experimental data of a single-stage fluidized bed and the axial dispersion model for the reason that they observed the movement of solids which occurred by either the gross solids movement or the diffusion effect under different gas velocities. And to get the flow model closely reflect the real situation, it is usually improved by means of various modified mathematical models or the correlating coefficients which lack scientific basis. So it is inadequate to accurately represent and completely analyze the RTD of solids mixing just by a simple functional model, absolutely.

Recently, with the rapid development of computational simulation technique, CFD has become a more and more powerful tool to investigate the complex fluidized flow [17–19]. As is known, the hydrodynamic properties of gas and solids have an important influence on the characteristics of solids RTD. So in order to completely study the function mechanism of flow field on RTD and accurately predict it, it is essential to analyze the RTD by means of CFD which could give much more information about the local values of solids holdup (ϵ_s) and their spatial

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distributions especially in the regions where measurements are either difficult or impossible. Therefore, it is extremely necessary to numerically investigate and precisely predict the RTD of multi-compartment fluidized bed system for its scale-up and optimization. But from the literature survey on the simulation of RTD by CFD summarized in Table 1, it can be obviously found that the traditional study pays more attention to the one-stage chemical reactor, but there has been merely no report on the calculation of the multi-compartment fluidized bed RTD until now.

The present work aims at the numerical simulation of the solids RTD in a multi-compartment fluidized bed by CFD. The effects of two-compartment bed structure, gas flow rate and bed outlet height on the RTD improvement had been determined, as well as the computed timed-averaged solids holdup and the flow structure were studied comprehensively.

2. Mathematical Modeling

2.1. Physical model

To improve the RTD of solids, a multi-compartment fluidized bed system was designed and investigated experimentally with positive results by Pongsivapai, Oregon State University in America [25]. Two fluidized beds were separated from each other by a segmentation baffle to make the solid flow pattern being apparently close to plug flow, where all particles would have a more similar residence time. The fresh solids were fed from the solid inlet of the first bed and discharged through the orifice to the second bed and overflowed out of the vessel from the outlet finally. After the fluidized system reached the steady state, the tracers were introduced into the bed instantaneously and all the particles started to collect at the exit orifice simultaneously. Glass beads with the mean diameter being 379 μm were used as the fluidized bed materials and their minimum fluidizing velocity (u_{mf}) was determined to be 0.161 $\text{m}\cdot\text{s}^{-1}$ experimentally. The experimental conditions of RTD measurement are listed in Table 2, and the detailed description of experimental apparatus, mechanism and plan can be found in the original thesis [25].

2.2. Numerical model

Present computation was conducted in accordance with Pongsivapai's procedures. The RTD and fluidized hydrodynamics of multi-compartment fluidized bed were simulated by using the Eulerian–Eulerian two-fluid

Table 2

Experimental conditions of RTD measurement

No.	$u_g/\text{m}\cdot\text{s}^{-1}$	h_{out}/m	$G_{\text{in}}/\text{kg}\cdot\text{s}^{-1}$	MRT, τ/min	Compartment of bed
1	0.208	0.1	0.00233	8.85	Single
2	0.162				
3	0.208				
4	0.305				
5	0.162	0.2	0.00205	16.18	Double
6	0.208				
7	0.305				

approach. The simplified kinetic theory of granular flow (KTGF) governing equations used for the CFD model are listed in Appendix. The current simulation was just performed in a 2D domain for the computing time saving, which is found to be accurate enough to predicate the RTD. In addition, for the numerations were carried out mainly for the dense section of the bed, so the grid interval spacing of the domains below and above the outlet height of 0.2 m was 5 and 10 mm, respectively. The computational domains with the boundary conditions are schematically displayed in Fig. 1, and the relevant parameter setting for the simulation is listed in Table 3. It should be noted that only one pair of boundaries of solid inlet and outlet conditions was set for either the height of 0.1 or 0.2 m in each computation, and the other pair was set to be the wall condition correspondingly.

2.3. Analysis method of RTD

As the most widely used RTD measurement, tracer technique was used to study the solid flow through fluidized bed [26–28]. For our simulation, the solids RTD was calculated by using the species transport model with Fluent 6.2.16, in which the tracer would be modeled as a species. Because the physical properties of tracer were identical with those of the solid material, the very low concentration of tracer would not have any significant effect on the flow field virtually. After the finite tracer particles were introduced into the fluidized system by setting the tracer mass fraction at the mass inlet equaling to 1 during one time step iteration, then the area-weighted-average concentration of the tracer at the outlet was monitored with time to obtain the RTD. The subsequent simulated flow time was at least four times of the MRT for each case, and the time-averaging of local values was performed within the above duration.

The tracer concentration of the sample $c(t)$ at any time t is:

$$c(t) = \frac{m_{\text{tracer}}}{m_{\text{tot}}} \quad (1)$$

Table 1

The numerical study of RTD for different chemical reactor by CFD

Author(s)	Research topic	Chemical reactor	Remarks
Han et al. [20]	Onset velocity of circulating fluidization and particle residence time distribution: A CFD–DEM study	CFB	The humps and trailing on the particle RTD curves suggest a complex micro-mixing behavior in liquid–solid systems originating from particle-scale behavior.
Andreuxa et al. [21]	Hydrodynamic and solids residence time distribution in a circulating fluidized bed: Experimental and 3D computational study	CFB	The numerical investigation of the solid mixing is deferred until later since the near-wall region where the solid phase down flow and mixing are predominant is not well predicted in spite of well-predicted vertical profiles of pressure.
Zhang et al. [22]	CFD simulation and experiment of residence time distribution in short-contact cyclone reactors	Cyclone reactors	The RTD characteristics were experimentally and numerically investigated. It indicates that there are vortex, backflow and other secondary flow in the reactor.
Bai et al. [23]	Modeling flow and residence time distribution in an industrial-scale reactor with a plunging jet inlet and optional agitation	Industrial-scale continuous liquid reactor	The RTD obtained is being used to improve the kinetic reactor model for prediction of reactor performance (yield and selectivity). The CFD simulations for the plunging jet coupled with the reactor flow fields and RTD in the reactor provide valuable insights for understanding hydraulic behavior, troubleshooting performance issues and optimizing reactor operations.
Li et al. [24]	Residence time distributions of different size particles in the spray zone of a Wurster fluid bed studied using DEM–CFD	Wurster fluid bed coater	The DEM–CFD simulations of different size particles show that large particles spend a longer time in the spray zone and in the Wurster tube than small particles.

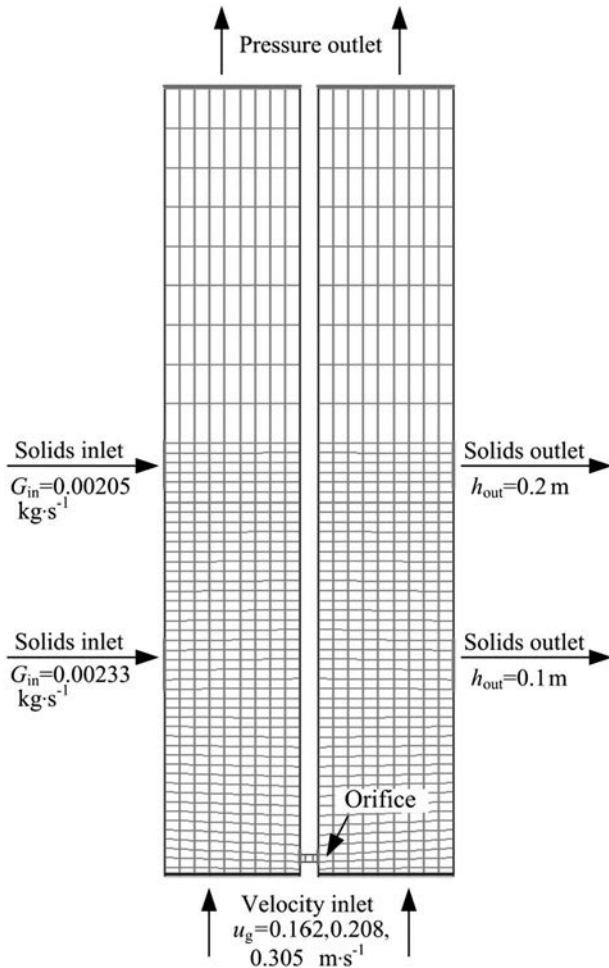


Fig. 1. The computational domain with boundary conditions for the multi-compartment fluidized bed.

Table 3
Parameters setting for the simulation

Description	Value
Diameter of particle, $d_p/\mu\text{m}$	379
Particle density, $\rho_p/\text{kg}\cdot\text{m}^{-3}$	2416
Gas density, $\rho_g/\text{kg}\cdot\text{m}^{-3}$	1.225
Gas viscosity, $\mu_g/\text{Pa}\cdot\text{s}$	1.7894×10^{-5}
Grid interval spacing, $\Delta z/\text{mm}$	5 or 10
Initial solid fraction, ε_i	0.60
Time interval, $\Delta t/\text{s}$	2×10^{-4}
Bed dimension, $W \times H/\text{m}^2$	0.1×0.4

The RTD of the total system can be obtained from the computed tracer concentration $c(t)$ as [26]:

$$E(t) = \frac{c(t)}{\int_0^\infty c(t)dt} \approx \frac{c(t_i)}{\sum_{i=0}^\infty c(t_i)\Delta t_i} \quad (2)$$

This is the most used of the distribution function connected with reactor analysis because it characterizes the lengths of time various solids spend within the reactor. Frequently, $E(t)$ function is normalized

with respect to the dimensionless time scale, $\theta = \frac{t}{\tau}$. Therefore, a dimensionless function $E(\theta)$ can be defined as:

$$E(\theta) = \tau E(t) \quad (3)$$

The other expression has been defined as the fraction of solids that has spent a time t or less than t , and it is called the cumulative residence time distribution and expressed as follows:

$$F(\theta) = \int_0^\theta E(\theta)d(\theta) \quad (4)$$

Additionally, the second momentum of the distribution curve (RTD variance, σ^2), indicates the degree of dispersion around the mean residence time (MRT), the larger the variance is, the higher the amplitude of the distribution is, and it is given by:

$$\sigma^2 = \int_0^\infty (t - \tau)^2 E(t)dt \quad (5)$$

And the relationship between the dispersion variance and the tanks-in-series model is [29]:

$$\sigma^2 = \frac{1}{N} \quad (6)$$

The third momentum of the distribution describes its skewness, s^3 , which is a measure of the asymmetry of the distribution. Negative value indicates a tail on the left side while positive skewness indicates a distribution with a longer tail on the right side, and is given by:

$$s^3 = \frac{\int_0^\infty (t - \tau)^3 E(t)dt}{\sigma^{3/2}} \quad (7)$$

3. Results and Discussion

3.1. Model validation

To validate the accuracy of the numerical simulation conducted in this study, the comparison of solid mass and mass flow rate through the orifice between the simulated and experimental results, and the computed recovery percent of tracers were obtained and evaluated, respectively.

Fig. 2 shows that the solid mass computed by the simulation is generally a little lower than the value measured experimentally in several conditions, and a reasonable agreement within deviation of $\pm 20\%$ is observed. This can be interpreted that besides the influence factors of gas velocity and bed outlet height, the computed solids mass is also mainly

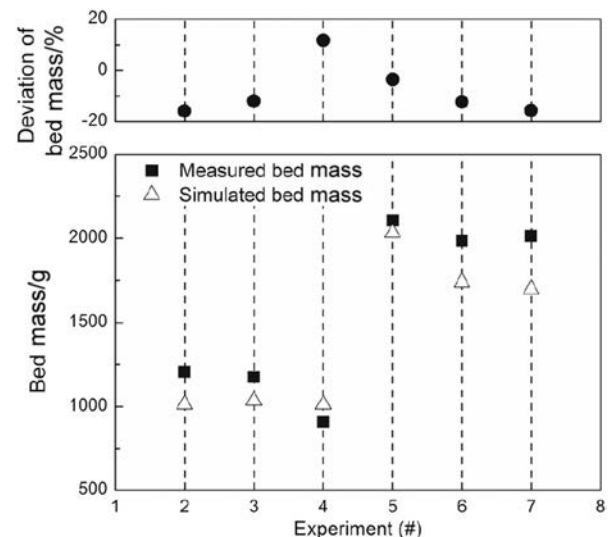


Fig. 2. The comparison of solid mass between the simulated and experimental results.

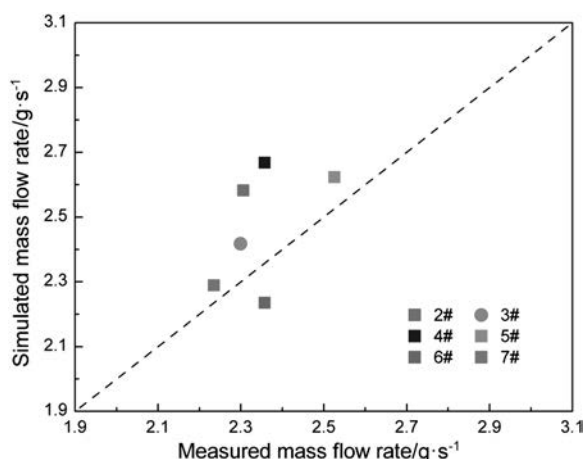


Fig. 3. The comparison of mass flow rate through the orifice between the simulated and experimental results.

dependent on the drag force between the solid–gas phases theoretically. For the adopted Gidaspow drag model generally overestimates the momentum exchange between phases and over predicts bed expansion further [19,30], so the simulated mass is less than the experimental value, correspondingly.

The diameter of orifice connecting the two adjacent fluidized bed compartments is 3.5 mm. The computed mass flow rate of solids

through the orifice is plotted against the experimental value in Fig. 3. For all the six conditions, the results of the calculation show few differences compared with the measured results available, where the deviation are less than 12%. It is found that the solid mass flow rate through the orifice is close to the feed rate of the bed when the system reaches the steady condition, this agrees well with the mechanical mass balance of the fluidization [31,32].

The computed percent of recovered tracers is plotted in Fig. 4. It is shown that the tracer recovery ranges from 94% to 104%, demonstrating that the computed method of RTD is reasonably correct. The above deviation is induced by either the dead region existing in the bed or the cumulative simulation error.

All the above comparisons prove that the values obtained by numerical simulation are accurate and reliable. And it is trustworthy to predicate the RTD and fluidized hydrodynamics for the multi-compartment fluidized bed based on these results.

3.2. Solids RTD

The computed solids RTD for the single- and multi-compartment fluidized beds with the same gas velocity for the experimental conditions of 1#, 3# and 6# are reflected in Fig. 5. It is indicated that the predicted RTD profiles of $E(t)$ are in reasonable agreement with experimental results. Compared with Fig. 5a and b, the RTD curve of solids improves substantially to the direction of plug flow when fluidized bed vessel is partitioned into two compartments, the RTD is narrowed and the location of main peak moves towards $t = \tau$ which implies that more particles would have the same residence time, all the above will be beneficial to overcoming the unfavorable back mixing and the possibility of bypassing for the fluidization [33]. With the MRT increasing, the RTD main peak intensity decreases and the tail extends longer, which is the typical RTD feature of bubbling fluidized bed as shown in Fig. 5c.

Fig. 6a–c and d–f shows the effect of fluidizing gas velocity on the solids RTD with the outlet height being 0.1 and 0.2 m, respectively. It is known that the mixing quality of fluidization depends on gas velocity, and it affects the RTD of solids further. Combined with Fig. 6 and Table 4, it is shown that with the gas velocity increasing, the RTD σ^2 rises and the number of tanks-in-series (N) decreases, correspondingly. This can be associated to the fact that when the fluidizing velocity increases beyond u_{mf} , the generated more gas phase enhances the fluid heterogeneity, the solids would fluidize smoothly and mix completely, thus, increasing the spread of residence time. In addition, with the outlet height rising, the RTD σ^2 also increases under three different gas velocities. This implies that the longer length the particles travel through the bed, the deeper the mixed flow tendency exhibit. The positive skewness, s^3 of $E(\theta)$ curves for all the experimental conditions displayed in Fig. 7 is corresponding to the longer tail on the right side of each RTD curve.

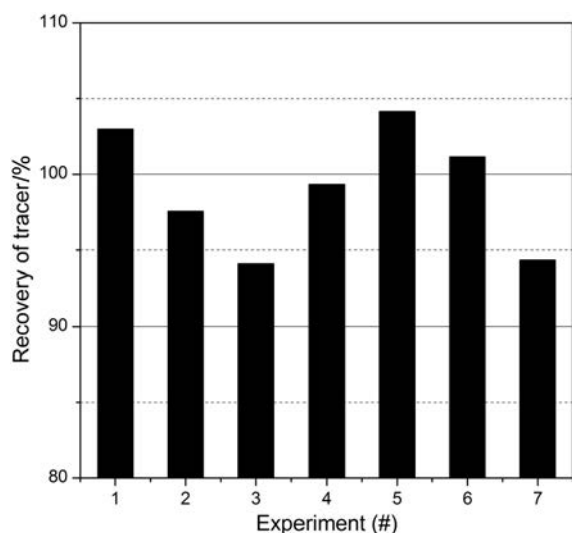


Fig. 4. The computed recovery percent of tracers.

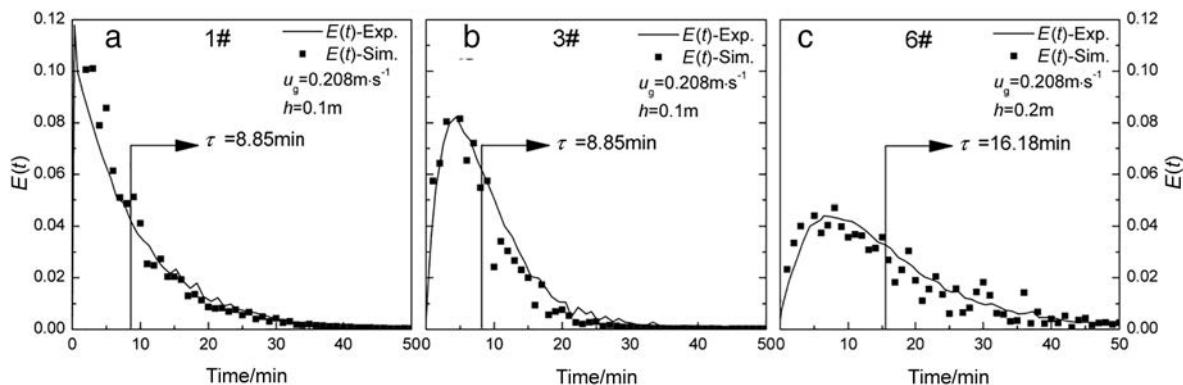


Fig. 5. The computed solids RTD for the single- and multi-compartment fluidized beds.

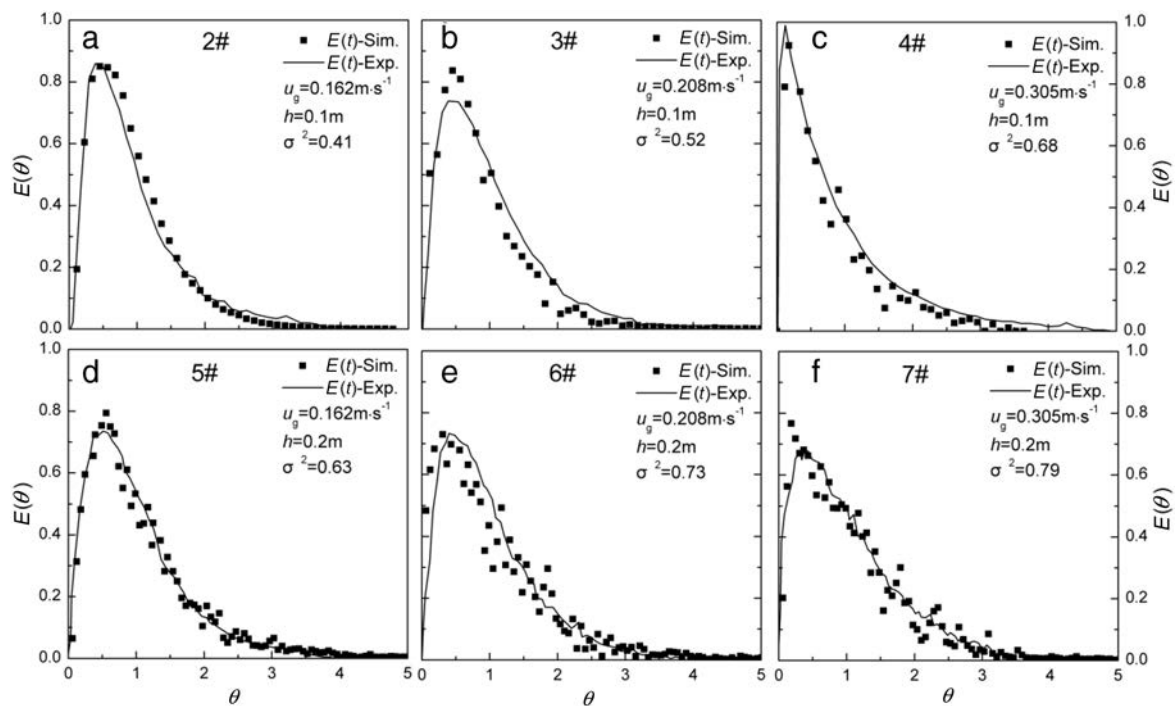


Fig. 6. The effect of fluidizing gas velocity on the solids RTD.

Table 4
The RTD σ^2 for all the experimental conditions

No.	$u_g/\text{m}\cdot\text{s}^{-1}$	h_{out}/cm	σ^2	N
1	0.208	100	0.88	1.1
2	0.162	100	0.41	2.4
3	0.208		0.52	1.9
4	0.305		0.68	1.5
5	0.162	200	0.63	1.6
6	0.208		0.73	1.4
7	0.305		0.79	1.3

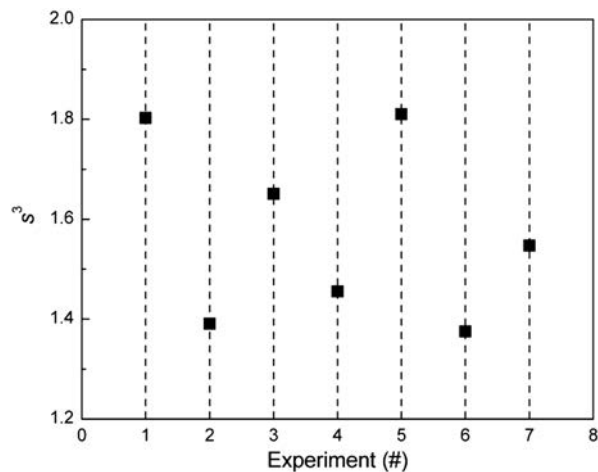


Fig. 7. The dimensionless skewness, s^3 of the $E(\theta)$ curves for all the experimental conditions.

3.3. Cumulative solids RTD

Fig. 8 presents the cumulative distribution of overall solids residence time for the different experimental conditions. It takes at least three

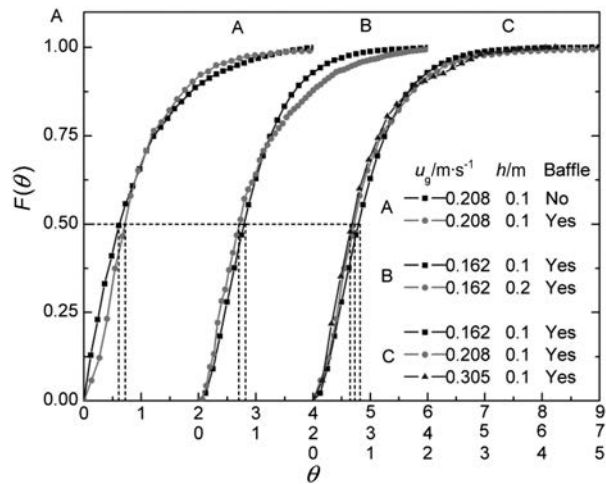


Fig. 8. The cumulative distribution of overall solids residence time.

times of MRT for all particles to flow out of the bed. To evaluate the effect of bed compartment structure (A), outlet height (B) and gas velocity (C) on RTD, t_{50} referring to the time of $F(\theta) = 0.5$ is introduced to character the relative residence time [34]. From the three groups comparison of A, B and C, it is revealed that the multi-compartment structure hinders the particles fluidized out of the bed with a longer t_{50} , but the solids relative residence time decreases with the bed height or the gas velocity rising. This can be explained that the baffle inserted into the multi-compartment bed increases the lateral flow resistance for the solids, while the bubble phase will expand and rise quickly with the increase of bed height or the gas velocity which promotes solids flow out of the bed faster, ultimately.

3.4. Computed radial profiles of solids holdup

To further investigate the influence of flow structure on the RTD, the radial profiles of time-averaged solids volume fraction for the multi-

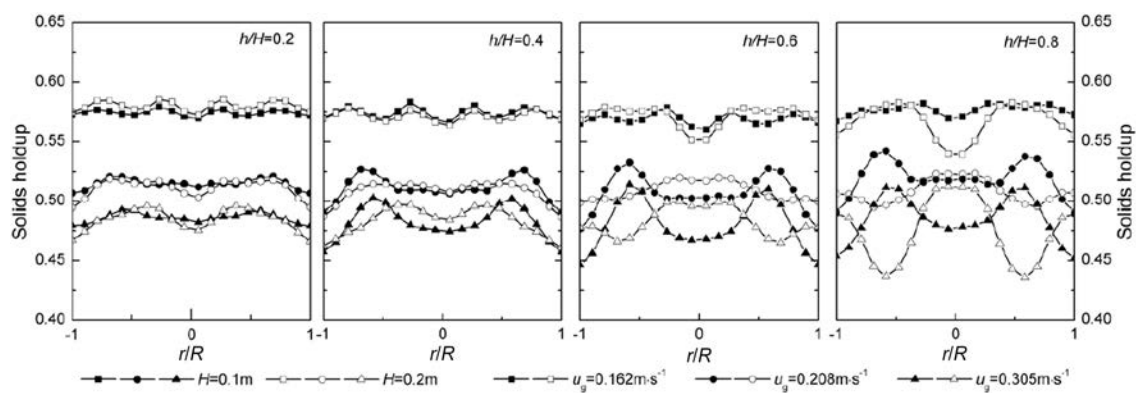


Fig. 9. The computed radial profiles of solids volume fraction for the multi-compartment fluidized bed with different outlet heights ($H = 0.1, 0.2$ m).

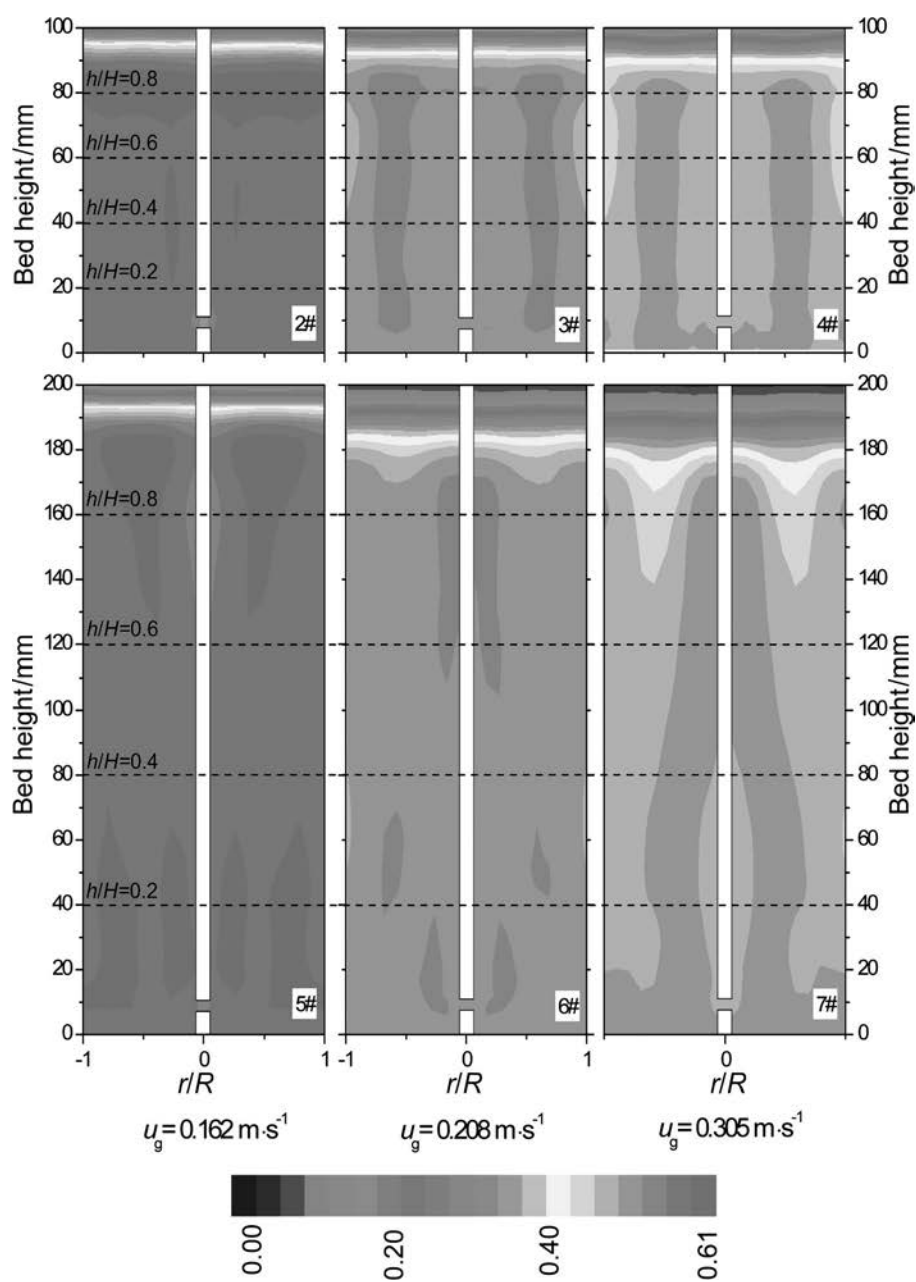


Fig. 10. The computed time-averaged solids holdup of the multi-compartment fluidized bed with different outlet heights ($H = 0.1, 0.2$ m).

compartment fluidized bed with different outlet heights ($H = 0.1, 0.2$ m) are computed and illustrated in Fig. 9. It is shown that the solids fluidize with a denser volume fraction ($\varepsilon_s = 0.43\text{--}0.57$) for the experimental gas velocity being just $1\text{--}1.9\ u_{mf}$ ($u_g = 0.162\text{--}0.305\ \text{m}\cdot\text{s}^{-1}$). Besides, it is found that with the bed height ratio (h/H) increasing, the solids volume fraction fluctuates more intensively, and the ε_s radial distribution of the two beds are more distinct from each. This can be interpreted that the bubble size increases with the elevation above the distributor, and the disturbance effect of bubble to the fluidized structure aggravates correspondingly. Even with the same h/H , the bubble of bed with $H = 0.2$ m is larger than that of $H = 0.1$ m which affects the fluidized structure further. And it is just the above heterogeneous fluidized structure that induces the widely non-uniform RTD, and different modifications have been proposed to improve the solids flow pattern to be close to plug flow [6,9–11,13].

3.5. Computed time-averaged solids holdup

From the time-averaged solids holdup contours of the multi-compartment fluidized bed shown in Fig. 10, it is revealed that the solids distributions of two bed vessels are nearly mirror to each other with the segmentation baffle as the symmetry axis. The global solids holdup and the height of dense bed decrease with the gas velocity rising, furthermore, the solids inventory would reduce that could be verified from the simulated and experimental bed mass illustrated in Fig. 2.

3.6. Simulated vector plots of solids velocity

In the end, the comparison of solids vector plots for the experimental conditions of 1#, 3# and 6# are illustrated in Fig. 11. It should be noted that the vector plots of solids velocity are produced from the instantaneous data, furthermore, Fig. 11 could represent the flow behavior of fluidized bed for it is selected from the numerous solids velocity maps at different flow times. Comparing Fig. 10a with b, it can be found that the baffle divides the individual internal circulation existing in the single fluidized bed into double ones, and it is just this modification that improves the RTD of solids flowing through the bed and makes all particles have a more similar residence time. Combined with the computed RTD σ^2 for different conditions listed in Table 4, it is reflected that raising the bed outlet height from $H = 0.1$ m (3#) to $H = 0.2$ m

(6#), the flow structure becomes more complex and the RTD σ^2 increases under the same gas velocity.

Further study is in progress to investigate and control the RTD for the particles with a wide size distribution and to the corresponding CFD simulations work.

4. Conclusions

The simulations of RTD and fluidized structure for multi-compartment fluidized bed were performed by using CFD. A general comparison of the numerical results with experimental data (solids mass and mass flow rate through the orifice) and the evaluation of the computed recovery percent of tracers show reasonable agreement. The approach of partitioning a fluidized bed into double vessels could reduce the gross back mixing and improve the solids RTD to the direction of plug flow with a relative longer residence time. With the fluidizing gas velocity or the bed outlet height rising, solids flow out of the bed more quickly with a wider spread of residence time and a larger RTD σ^2 . The computed radial profiles of solids holdup fluctuate more intensively with h/H increasing, which leads to the heterogeneous fluidized structure and induces the widely non-uniform RTD. From the global distributions of solids holdup and velocity in the two bed vessels, it is also revealed that the division of the individual internal circulation into double ones improves the flow pattern to be close to plug flow, and the flow structure becomes more complex with the bed outlet height rising.

Nomenclature

$\bar{C}_D$	drag coefficient, dimensionless
$c(t)$	concentration of the tracer at time of t , dimensionless
D_b	bubble diameter, m
d_p	particle diameter, μm
$E(t)$	residence time distribution function, min^{-1}
$E(\theta)$	dimensionless residence time distribution function, dimensionless
F_d	the total drag force on the particles in a unit volume of bed, $\text{N}\cdot\text{m}^{-3}$
$F(\theta)$	cumulative dimensionless residence time distribution function, dimensionless
G_{in}	feed rate of solids, $\text{kg}\cdot\text{s}^{-1}$
g	gravitational acceleration constant, $\text{m}\cdot\text{s}^{-2}$
H	outlet height, m

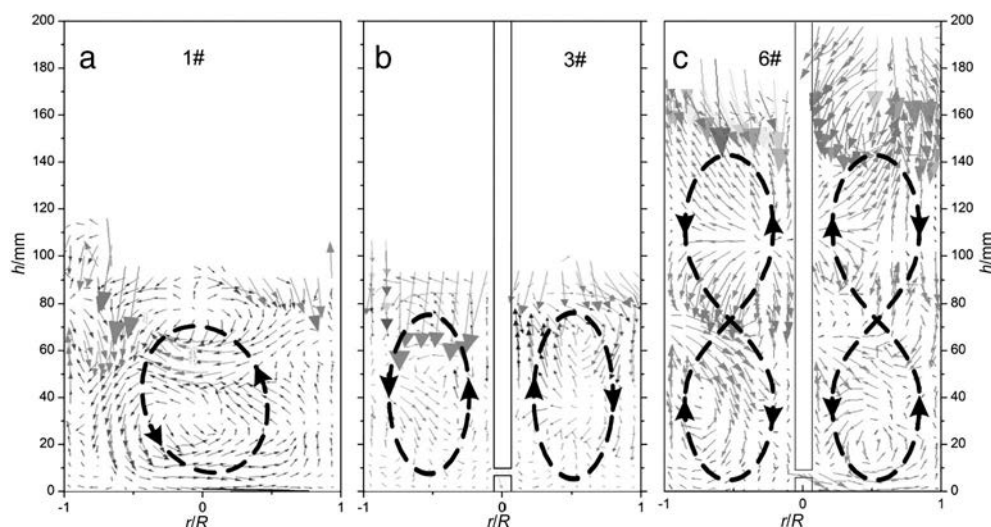


Fig. 11. Comparison of solids vector plots for the experimental conditions of 1#, 3# and 6#.

h	height above air distributor, m
m	mass, kg
N	number of ideal stirred tanks, dimensionless
s^3	skewness of the residence distribution curve, dimensionless
t	time, min
$\hat{t}$	computed average residence time, min
u	real velocity, $\text{m} \cdot \text{s}^{-1}$
β	drag coefficient, $\text{kg} \cdot \text{m}^{-3} \cdot \text{s}^{-1}$
Δt	time interval, s
Δz	grid interval spacing, mm
δ	the deviation, dimensionless
ε	phase holdup, dimensionless
θ	dimensionless time, dimensionless
μ_g	gas viscosity, $\text{Pa} \cdot \text{s}$
ρ_p	solid density, $\text{kg} \cdot \text{m}^{-3}$
ρ_g	gas density, $\text{kg} \cdot \text{m}^{-3}$
σ^2	variance of the residence distribution curve, dimensionless
τ	experimental mean residence time, min

Subscripts

g	gas phase
i	control variable
m	mass
out	outlet
p	solids particle
t	time
tot	total
t_c	the calculated tracer's property

Appendix. The simplified kinetic theory of granular flow (KTGF) governing equations used for the CFD model

Table A1

Governing equations for two-fluid model

Continuity equation ($k = g, s$)	
$\frac{\partial}{\partial t}(\varepsilon_k \rho_k) + \nabla \cdot (\varepsilon_k \rho_k \vec{u}_k) = 0$	
Momentum equation	
$\frac{\partial}{\partial t}(\varepsilon_g \rho_g \vec{u}_g) + \nabla \cdot (\varepsilon_g \rho_g \vec{u}_g \vec{u}_g)$	
$= -\varepsilon_g \nabla p + \varepsilon_g \rho_g \vec{g} + \nabla \cdot \bar{\tau}_g + \beta(\vec{u}_s - \vec{u}_g)$	
$\frac{\partial}{\partial t}(\varepsilon_s \rho_s \vec{u}_s) + \nabla \cdot (\varepsilon_s \rho_s \vec{u}_s \vec{u}_s)$	
$= -\varepsilon_s \nabla p - \nabla p_s + \varepsilon_s \rho_s \vec{g} + \nabla \cdot \bar{\tau}_s + \beta(\vec{u}_g - \vec{u}_s)$	
Stress-strain tensor for gas and solid phases ($k = g, s$)	
$\bar{\tau} = \varepsilon_k \mu_k (\nabla \vec{u}_k + \nabla \vec{u}_k^T) - \frac{2}{3} \varepsilon_k \mu_k (\nabla \cdot \vec{u}_k) \vec{I}$	
Solid pressure	
$p_s = \varepsilon_s \rho_s \Theta_s + 2\varepsilon_s^2(1 + e)\rho_s g_0 \Theta_s$	
Solid phase shear viscosity	
$\mu_s = \mu_{s, \text{kin}} + \mu_{s, \text{col}} + \mu_{s, \text{fr}}$	
$\mu_{s, \text{kin}} = \frac{5\rho_s d_s \sqrt{\pi\Theta_s}}{48\varepsilon_s(1+e)g_0} [1 + \frac{4}{3}g_0\varepsilon_s(1+e)]^2$	
$\mu_{s, \text{col}} = \frac{4}{5\sqrt{\pi}} \varepsilon_s \rho_s g_0 d_s (1+e) \sqrt{\Theta_s}$	
$\mu_{s, \text{fr}} = \frac{p_s \sin\phi}{2\sqrt{I_{2D}}}$	
Solid phase bulk viscosity	
$\lambda_s = \frac{4}{3\sqrt{\pi}} \varepsilon_s \rho_s g_0 d_s (1+e) \sqrt{\Theta_s}$	
Radial distribution function	
$g_0 = [1 - (\frac{\varepsilon_s}{\varepsilon_{s, \text{max}}})^{1/3}]^{-1}$	
Inter-phase drag coefficient (Gidaspow drag model)	
$\beta = \begin{cases} \frac{3}{4} C_D \frac{\rho_g \varepsilon_g \vec{u}_s - \vec{u}_g }{d_s} \varepsilon_g^{-2.65} & (\varepsilon_g > 0.8) \\ 150 \frac{\varepsilon_g^2 \mu_g}{\varepsilon_g d_s^2} + 1.75 \frac{\varepsilon_g \rho_g \vec{u}_s - \vec{u}_g }{d_s} & (\varepsilon_g \leq 0.8) \end{cases}$	
$C_D = \begin{cases} \frac{24}{Re} (1 + 0.15 Re^{0.687}) & (Re \leq 1000) \\ 0.44 & (Re > 1000) \end{cases}$	
$Re = \frac{\varepsilon_g \rho_g d_s \vec{u}_g - \vec{u}_s }{\mu_g}$	

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